Exact Methods and Heuristic Approaches for Setup Minimization of One-Dimensional Cutting Stock Problems

Master Thesis

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Problem Statement

In the pulp and paper-, metal- and plastic film industry, effective solution techniques for frequently occurring blending and cutting stock problems are of great economical and environmental interest. The latter is the problem of cutting standardized paper rolls (from the point of the pulp and paper industry, see Figure 1) into quantities (demands) of smaller rolls (items/order widths) being requested by customers, while minimizing the material usage, *i. e.*, the quantities of paper rolls being cut resp. while minimizing the waste (cutoff reduction), leading to cost- and resource-saving, environment-friendly production processes. This problem involves the construction of cutting patterns while determining the application quantities on the paper rolls for each pattern. However, it does not consider the costs when changing from one pattern to another (setup costs) by re-adjusting the knife arrangement. Reducing the setup costs by using as few different patterns as possible is known as setup minimization problem in literature. The demand levels have to be fulfilled exactly.

Figure 1: Master roll in the paper industry

Example

There are four order widths $\mathcal{I} = \{i_1, i_2, i_3, i_4\}$ with width w_i and demand levels D_i, summarized in the table below. The width of the master rolls is given by $W = 1000$, the number of knifes is $K = 7$.

i	w_i	D_i
i_1	100	60
i_2	200	30
i_3	500	20
i_4	300	10

At first, we will calculate a lower bound on the minimal number z^{opt} of different patterns. Since not all order widths can be combined in a single pattern, *i. e.*, $\sum_{i \in \mathcal{I}} w_i = 1100 > 1000 = W$, we obviously need more than a single pattern in order to fulfil the demand for all order widths (exactly). However, two patterns are probably sufficient. We will now construct a solution with two different patterns by looking closely.

Obviously, one can combine the order widths in several ways (two of them presented below), each using two patterns. Since two is an lower bound on z^{opt}, both solutions are optimal with respect to the number of patterns (minimal number of setups).

	p_1	p_2	
u_p	20	10	D_i
$w_1 = 100$	3	0	60
$w_2 = 200$	0	3	30
$w_3 = 500$	1	0	20
$w_4 = 300$	0	1	10
width	800	900	
cutoff	200	100	

	p_1	p_2	
u_p	20	10	D_i
$w_1 = 100$	2	2	60
$w_2 = 200$	1	1	30
$w_3 = 500$	1	0	20
$w_4 = 300$	0	1	10
width	900	700	
cutoff	100	300	

For instance, the first pattern p_1 of the solution on the left will be applied 20 times, *i. e.*, 20 master rolls are cut according to pattern p_1. In particular, we will cut the first order i_1 three times and the third order i_3 a single time out of the master roll. The width of the master roll and the number of knifes is not exceeded, while the demand levels of all order widths are fulfilled exactly.

Contributions

We present several novel exact linear model formulations for the setup minimization problem with exact demand fulfilment. Based on these formulations, instances of low or medium complexity can be solved to optimality in reasonable time. In order to obtain a general performance improvement, we use binary variables only, while reducing the resulting high number of variables by exploiting problem specific properties, *e. g.*, we apply tighter bounds on the pattern multiplicities being derived by the demand levels, leading to a significantly reduce in terms of solution time. A further benefit besides the fast implementability with an algebraic modeling language is the easy adaptability of our models, *i. e.*, additional requirements can be included straightforward (*e. g.*, diverging objectives or further constraints).

In order to solve problem instances with increasing complexity, we present several novel and easy to implement heuristic approaches. These approaches are characterized by a transparent top-level conception. In particular, we are repeatedly solving minor complex models by the comfortable usage of an algebraic modeling language (`GAMS`) combined with an optimization solver (`CPLEX`). For instance, a greedy algorithm being proposed generates in each iteration a pattern which maximizes the number of orders being fulfilled exactly, while also observing the already generated patterns within the decision process, as one might add further widths to those patterns. This task will be realized best and most comfortable by formulating an appropriate model (which is of low complexity) being solved in each iteration to optimality.

As the requirement of pure setup minimization with exact demand fulfilment often leads to solutions with high material consumption, most of the approaches will be modified in order to observe the minimization of material consumption as secondary objective, too. Table 1 summarizes the different approaches being developed in this thesis.

Next to an evaluation of our approaches based on numerical results calculated on a broad range of problem instances frequently used in literature, we also give some theoretical insights in relation with our approaches, *e. g.*, we prove upper and lower bounds on the pattern multiplicity or that it is sufficient to solve the relaxed variant of a model.

Approach	Short Description	Exact
`PG`	Model for Pattern Generation (using the `CPLEX Solution Pool` Option)	-
`BLMPG`	Binary Linear Model based on complete Pattern Generation	✓
`BLMPG-NR1`	Solving `BLMPG` initially, reducing the Number of Master Rolls subsequently	✓
`BLMPG-NR2`	Observing the Number of Patterns and Master Rolls in the objective of `BLMPG`	✓
`BLM`	Binary Linear Model for Setup Minimization with exact Demand Fulfilment	✓
`BLM-NR1`	Solving `BLM` initially, reducing the Number of Master Rolls subsequently	✓
`BLM-NR2`	Observing the Number of Patterns and Master Rolls in the objective of `BLM`	✓
`BLMEP`	Model based on generated Efficient Patterns combined with free Patterns	✓
`BLMFV`	Binary Linear Model similar to `BLM` but with Fewer Variables	✓
`BLMPG-MMR`	Formulation similar to `BLMPG`, but observing multiple Types of Master Rolls	✓
`BLM-MMR`	Formulation similar to `BLM`, but observing multiple Types of Master Rolls	✓
`Greedy1`	Greedy Algorithm based on the `G1A` Model	✗
`Greedy2`	Greedy Algorithm based on the `G2A` Model	✗
`Greedy3`	Greedy Algorithm based on the `G3A` Model	✗
`Greedy3*`	Greedy Algorithm based on the `G3A` Model including cutoff reduction	✗
`DDA`	Demand Dividing Algorithm	✗
`DDA-NR1`	Demand Dividing Algorithm including cutoff reduction	✗
`G3-BLM`	Combining `Greedy3` with the `BLM` formulation	✓
`DDA-BLM`	Combining `DDA` with the `BLM` formulation	✓

Table 1: Overview on the different Approaches

Exact Model Formulation

In the following, we want to present one of the monolithic model formulation (referred as `BLM`). At first, we have to introduce some notation:

- Set of patterns $p \in \mathcal{P}$.
- Set of pattern multiplicities $k \in \mathcal{K} := \{1, \ldots, \max_{i \in \mathcal{I}} D_i\}$ and the set of item multiplicities $n \in \mathcal{N} := \{1, \ldots, \min[\max_{i \in \mathcal{I}} \lfloor W/w_i \rfloor ; K]\}$.
- Two kinds of binary variables: δ_{kp} indicates whether p is used exactly k times and γ_{iknp} indicates whether p contains i exactly n times and is used exactly k times.

The objective is to minimize the number of different patterns (see model `BLM` on the right). Exact demand fulfilment is enforced by the first constraint (note the linear term). The width of the master roll and the number of knifes is observed by the second and third constraint. Also note that we use these constraints to connect the variables δ_{kp} and γ_{iknp} appropriately. The fourth constraint enforces that the next pattern $p+1$ will only be used, if pattern p is already used, while constraint five enforces the pattern multiplicities to be sorted in descending order (symmetry breaking constraints). Both constraints tighten up the solution space to reduce the solution time. The model is completed by ensuring the uniqueness of the item multiplicity n of width i resp. the multiplicity k of a pattern p.

Binary **L**inear **M**odel - `BLM`:

$$\min \; z = \sum_{p \in \mathcal{P}} \sum_{k \in \mathcal{K}} \delta_{kp}$$

$$\begin{aligned}
\sum_{p \in \mathcal{P}} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} kn\gamma_{iknp} &= D_i && \forall i \in \mathcal{I} \\
\sum_{i \in \mathcal{I}} \sum_{n \in \mathcal{N}} w_i n\gamma_{iknp} &\leq W\delta_{kp} && \forall (k,p) \in \mathcal{K} \times \mathcal{P} \\
\sum_{i \in \mathcal{I}} \sum_{n \in \mathcal{N}} n\gamma_{iknp} &\leq K\delta_{kp} && \forall (k,p) \in \mathcal{K} \times \mathcal{P} \\
\sum_{k \in \mathcal{K}} \delta_{k,p+1} &\leq \sum_{k \in \mathcal{K}} \delta_{kp} && \forall p \in \mathcal{P} \setminus \{p_{|\mathcal{I}|}\} \\
\sum_{k \in \mathcal{K}} k\delta_{k,p+1} &\leq \sum_{k \in \mathcal{K}} k\delta_{kp} && \forall p \in \mathcal{P} \setminus \{p_{|\mathcal{I}|}\} \\
\sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} \gamma_{iknp} &\leq 1 && \forall (i,p) \in \mathcal{I} \times \mathcal{P} \\
\sum_{k \in \mathcal{K}} \delta_{kp} &\leq 1 && \forall p \in \mathcal{P}
\end{aligned}$$

Results

Our approaches have been tested on a broad range of benchmark instances used in literature, including 80 real-world instances from industrial applications as well as 1980 randomly generated `CUTGEN` instances. Table 2 summarizes numerical results (average number of patterns $\overline{z}$ and the average computation time) of three approaches - among them the results for the model formulation `BLM` presented above - on a subset of all instances.

By solving the monolithic model formulation, we are able to calculate and prove optimal solutions for all real-world instances. For some of these instances, no optimal solutions have been known so far. The most stable heuristic (`Greedy3`) being proposed provides solutions within seconds on average, being only 13 % worse in terms of pattern count compared to the optimal solutions on average (resp. by 1.19 patterns in absolute terms). This evaluation is based on 1060 instances being solved to optimality - for the first time in literature - by applying the monolithic model formulations. Compared to the optimal solutions for the instances from the industrial applications, these heuristic solutions are in general at most one or two patterns worse (on average by 0.8 patterns).

	-BLM-		-Greedy3-		-DDA-	
Class(#Inst.)	$\overline{z}$	$\overline{\text{sec}}$	$\overline{z}$	$\overline{\text{sec}}$	$\overline{z}$	$\overline{\text{sec}}$
Kallrath(25)	**7.36**	2790	8.12	3	8.00	4
Vanderbeck(16)	**6.13**	3669	7.19	4	7.25	5
Fiber(39)	**4.51**	314	5.23	3	5.49	2
type01(100)	**3.11**	4	3.80	1	3.64	2
type03(100)	**4.56**	58	5.74	2	5.61	6
type04(100)	—	—	8.47	28	8.09	68
type08(100)	—	—	9.00	4	11.25	3
type09(100)	**11.72**	71	13.16	3	14.98	7
type15(100)	**14.31**	41	15.27	3	17.54	5
bel20_1(20)	—	—	13.85	7	16.8	13
bel50_4(20)	**20.85**	868	25.25	7	20.95	553
bel150_9(20)	—	—	80.30	259	—	—

* All entries are arithmetic means. Optimal values are **bold**.

Table 2: Results for a subset of all instances and methods

Forthcoming Research

We plan to adapt the approaches in order to solve the one-dimensional cutting stock problem with setup minimization and exact demand fulfilment while observing master rolls of varying width (*e. g.* small, medium and large sized master rolls).